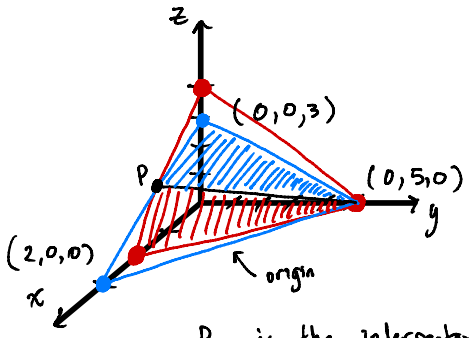
Math 354 - Spring 2024
Participation #2 Name: Solutions netID: _____

1. Consider the linear programming problem of the form: Maximize $z = \mathbf{c}^\top \mathbf{x}$, subject to $A\mathbf{x} \leq \mathbf{b}$ and $\mathbf{x} \geq \mathbf{0}$, where

$$A = \begin{bmatrix} 5 & 3 & 5 \\ 10 & 4 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 15 \\ 20 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

Draw a sketch of the set of feasible solutions to the problem. Identify and label each of the extreme points of the region (if any exist).



$5x + 3y + 5z = 15$
$(3,0,0)$ $(0,5,0)$ $(0,0,3)$

$10x + 4y + 5z = 20$
$(2,0,0)$ $(0,5,0)$ $(0,0,4)$

P is the intersection of planes: $10x+4y+5z = 20$, $5x+3y+5z = 15$, $y = 0$

$x = 1$ $y = 0$ $z = 4$

$$\left[\begin{array}{ccc|c} 10 & 5 & & 20 \\ 5 & 5 & & 15 \end{array}\right]$$
$$\to \left[\begin{array}{ccc|c} 2 & 1 & & 4 \\ 1 & 1 & & 3 \end{array}\right] \begin{matrix} \frac{1}{5}R_1 \\ \frac{1}{5}R_2 \end{matrix}$$
$$\to \left[\begin{array}{ccc|c} 0 & -1 & & -2 \\ 1 & 1 & & 3 \end{array}\right] R_1 - 2R_2$$
$$\to \left[\begin{array}{ccc|c} 0 & -1 & & -2 \\ 1 & 0 & & 1 \end{array}\right] R_2 + R_1$$

The set of extreme points is $\{(0,0,0), (2,0,0), (0,5,0), (0,0,3), (1,0,4)\}$

Note: The set of feasible solutions consists of all convex combinations of the extreme points.