

1. Each of the following linear programming models is neither standard nor canonical. Explain (all of the ways) why.

- (a) Minimize $z = 3x + 2y$ subject to the constraints

not standard/canonical because it is a minimization problem.

$$\begin{aligned} 2x + y &\leq 4 \\ 3x - 2y &\leq 6 \\ x \geq 0, y &\geq 0. \end{aligned}$$

Not canonical because these are $\leq$

- (b) Maximize $z = 2x_1 + 3x_2 + 4x_3$ subject to the constraints

$$\begin{aligned} 3x_1 + 2x_2 - 3x_3 &\leq 4 \\ 2x_1 + 3x_2 + 2x_3 &\leq 6 \\ 3x_1 - x_2 + 2x_3 &\geq -8 \\ x_1 \geq 0, x_2 \geq 0, x_3 &\geq 0. \end{aligned}$$

not standard because need all $\leq$
not canonical because need all $=$

- (c) Maximize $z = 3x + 2y + 3v - 2w$ subject to the constraints

$$\begin{aligned} 2x + 6y + 2v - 4w &= 7 \\ 3x + 2y - 5v + w &= 8 \\ 6x + 7y + 2v + 5w &\leq 4 \\ x \geq 0, y \geq 0, v \geq 0, w &\geq 0. \end{aligned}$$

need this to be $=$ in order to be canonical

need those to be $\leq$ in order to be standard.

- (d) Maximize $z = 2x + 5y$ subject to the constraints

$$\begin{aligned} 3x + 2y &\leq 6 \\ 2x + 9y &\leq 8 \\ x &\geq 0. \end{aligned}$$

Not standard (or canonical) because the y-variable is unconstrained.

Need equalities to be canonical.

need $y \geq 0$ for both standard and Canonical

- (e) Minimize $z = 2x_1 + 3x_2 + x_3$ subject to the constraints

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 4 \\ 3x_1 + 2x_2 + x_3 &= 8 \\ x_1 - x_2 &= 6 \\ x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

Not canonical because x_3 is unconstrained

Need $\leq$ to be standard

2. Use the techniques discussed in class to turn the linear programming problems from the previous page into standard (and canonical) form.

(a) Replace Minimize $z = 3x + 2y$ we Maximize $z = -3x - 2y$

To make it canonical form replace $\begin{matrix} 2x + y \leq 4 \\ 3x - 2y \leq 6 \end{matrix}$ with $\begin{matrix} 2x + y + u = 4 \\ 3x - 2y + v = 6 \\ u \geq 0, v \geq 0 \end{matrix}$

(b) To convert to standard form, replace $3x_1 - x_2 + 2x_3 \geq -8$ with $-3x_1 + x_2 - 2x_3 \leq 8$

To convert to canonical form, replace $\begin{matrix} 3x_1 + 2x_2 - 3x_3 \leq 4 \\ 2x_1 + 3x_2 + 2x_3 \leq 6 \\ -3x_1 + x_2 - 2x_3 \leq 8 \end{matrix}$ with $\begin{matrix} 3x_1 + 2x_2 - 3x_3 + x_4 = 4 \\ 2x_1 + 3x_2 + 2x_3 + x_5 = 6 \\ -3x_1 + x_2 - 2x_3 + x_6 = 8 \\ x_4 \geq 0, x_5 \geq 0, x_6 \geq 0 \end{matrix}$

(c) To convert to standard form, replace $\begin{matrix} 2x + 6y + 2v - 4w = 7 \\ 3x + 2y - 5v + w = 8 \end{matrix}$ with $\begin{matrix} 2x + 6y + 2v - 4w \leq 7 \\ -2x - 6y - 2v + 4w \leq -7 \\ 3x + 2y - 5v + w \leq 8 \\ -3x - 2y + 5v - w \leq -8 \end{matrix}$

To convert to standard form, leave $\begin{matrix} 2x + 6y + 2v - 4w = 7 \\ 3x + 2y - 5v + w = 8 \end{matrix}$ (and other stuff) as is,

and replace $6x + 7y + 2v + 5w \leq 4$ with $6x + 7y + 2v + 5w + z = 4$
 $z \geq 0$

(d) To convert to standard form, replace the unconstrained variable y with $y^+ - y^-$
with $y^+ \geq 0, y^- \geq 0$. Then the inequalities $\begin{matrix} 3x + 2y \leq 6 \\ 2x + 9y \leq 8 \end{matrix}$ become $\begin{matrix} 3x + 2y^+ - 2y^- \leq 6 \\ 2x + 9y^+ - 9y^- \leq 8 \end{matrix}$

To convert to canonical form, we further replace $\begin{matrix} 3x + 2y^+ - 2y^- \leq 6 \\ 2x + 9y^+ - 9y^- \leq 8 \end{matrix}$ with

$$\begin{matrix} 3x + 2y^+ - 2y^- + z = 6 \\ 2x + 9y^+ - 9y^- + w = 8 \\ z \geq 0, w \geq 0 \end{matrix}$$

(e) To convert to canonical form we must replace the unconstrained variable x_3 with x_3^+ and x_3^- . Then $\begin{matrix} 2x_1 + x_2 - x_3 = 4 \\ 3x_1 + 2x_2 + x_3 = 8 \\ x_1 - x_2 = 6 \end{matrix}$ becomes $\begin{matrix} 2x_1 + x_2 - x_3^+ + x_3^- = 4 \\ 3x_1 + 2x_2 + x_3 - x_3^- = 8 \\ x_1 - x_2 = 6 \end{matrix}$
 $x_3^+ \geq 0, x_3^- \geq 0$

Starting with this new situation, to get to standard form $2x_1 + x_2 - x_3^+ + x_3^- = 4$ gets replaced with $\begin{matrix} 2x_1 + x_2 - x_3^+ + x_3^- \leq 4 \\ -2x_1 - x_2 + x_3^+ - x_3^- \leq 4 \end{matrix}$, and similarly deal with $3x_1 + 2x_2 + x_3 - x_3^- = 8$.