Math 354
Quiz #3 Name: Solutions netID: _____ Section: _____

1. Consider the linear programming problem with constraints $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x} \geq \mathbf{0}$ where

$$A = \begin{bmatrix} 2 & 1 & 3 & 1 & 0 & 0 \\ 4 & 3 & 1 & 0 & 1 & 0 \\ -1 & 2 & 4 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 6 \\ 3 \\ 8 \end{bmatrix},$$

and $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^\top$. Let $\mathbf{z} = \begin{bmatrix} \frac{3}{10} & 0 & \frac{9}{5} & 0 & 0 & \frac{11}{10} \end{bmatrix}^\top$.

(a) Verify that $\mathbf{z}$ is a feasible solution.

To be feasible, we need $A\vec{z} = \vec{b}$, and $\vec{z} \geq \vec{0}$

$$A\vec{z} = \frac{3}{10}\begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} + \frac{9}{5}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} + \frac{11}{10}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 6+54+0 \\ 12+18+0 \\ -3+72+11 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 8 \end{bmatrix} \checkmark$$

$\vec{z} \geq \vec{0}$ ✓ . Therefore, $\vec{z}$ is feasible.

(b) Determine whether or not $\mathbf{z}$ is a *basic* feasible solution. Explain/justify your reasoning. If yes, what are the basic variables?

For $\vec{z}$ to be basic, we need that at least 3 of the components of $\vec{z}$ are zero. This is certainly true since $z_2 = z_4 = z_5 = 0$. We also need that the basic columns of $A$ (in this case $A_1, A_3, A_6$) be linearly independent.

$$[A_1\ A_3\ A_6] = \begin{bmatrix} 2 & 3 & 0 \\ 4 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix} \sim \begin{bmatrix} -1 & 4 & 1 \\ 4 & 1 & 0 \\ 2 & 3 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \sim \begin{bmatrix} 1 & -4 & -1 \\ 4 & 1 & 0 \\ 2 & 3 & 0 \end{bmatrix} \xrightarrow{-R_1}$$

$$\sim \begin{bmatrix} 1 & -4 & -1 \\ 0 & 17 & 4 \\ 0 & 11 & 2 \end{bmatrix} \begin{matrix} R_2 - 4R_1 \\ R_3 - 2R_1 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{The columns } A_1, A_3, A_6 \text{ are linearly independent.}$$

↑ Since these are linearly independent ⇑

Therefore $\vec{z}$ is basic.

2. The following tableau came up in the process of using the simplex method to solve a linear programming problem.

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | |
|---|---|---|---|---|---|---|
| $x_4$ | 0 | 4 | 0 | 1 | 6 | 15 |
| $x_1$ | 1 | 2 | 0 | 0 | −3 | 6 |
| $x_3$ | 0 | 2 | 1 | 0 | 4 | 17 |
| | 0 | −2 | 0 | 0 | −1 | 14 |

θ-ratios

$15/4 = 3.75$

$6/2 = 3$

$17/2 = 8.5$

(a) Determine the pivot row and column for this tableau (and show your justification) and complete ~~***one step*** of the simplex method~~.

Pivot column is $x_2$ since $-2$ is most negative value in the objective row.

Pivot row is $R_2$ since this has the least non-negative θ-ratio.

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | |
|---|---|---|---|---|---|---|
| $x_4$ | $-2$ | 0 | 0 | 1 | 12 | 3 |
| $x_1$ | $\frac{1}{2}$ | 1 | 0 | 0 | $-\frac{3}{2}$ | 3 |
| $x_2$ | $-1$ | 0 | 1 | 0 | 7 | 14 |
| | 0 | 0 | 1 | 0 | $-4$ | 31 |

$R_1^{old} - 2R_2^{old}$

$\frac{1}{2}R_2^{old}$

$R_3^{old} - R_2^{old}$

$R_4^{old} + R_2^{old}$

(b) Answer the following questions about your new tableau.

i. What is the corresponding basic feasible solution?

$(3, \ , 0, \ , 0, 0)$

ii. Is it optimal? (Why or why not?)

No because objective row still has negative value

iii. What is the value of the objective function at this feasible solution?

$Z = 31.$