

Consider the linear programming problem to maximize $z = 4x_1 + 2x_2 + 7x_3$ subject to

$$2x_1 - x_2 + 4x_3 \leq 18$$

$$4x_1 + 2x_2 + 5x_3 \leq 10$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

1. Transform this problem into one in canonical form: Maximize $z = 4x_1 + 2x_2 + 7x_3$ subject to $A\mathbf{x} = \mathbf{b}$. (Hint: A will be a 2×5 matrix.)
2. Determine which subsets $I \subseteq \{1, 2, 3, 4, 5\}$ (with $\#I = 2$) have the property that the columns A_i for $i \in I$ are linearly independent. For each one, find the corresponding basic solution. Is it feasible? (Use the back side of this sheet, as necessary.)
3. Find the optimal solution.