

Complete the following as instructed.

1. Let

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix},$$

- (a) Find the reduced row echelon form of  $[A : I_3]$ . Clearly identify at each step all of the elementary row operations applied.

$$\begin{aligned} \left[ \begin{array}{ccc|ccc} 0 & 2 & 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ -1 & 2 & 3 & 0 & 0 & 1 \end{array} \right] &\rightarrow \left[ \begin{array}{ccc|ccc} 0 & 1 & 0 & \frac{1}{2} & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 5 & 5 & 0 & 1 & 1 \end{array} \right] \begin{array}{l} \frac{1}{2}R_1 \\ R_3 + R_2 \end{array} \\ &\rightarrow \left[ \begin{array}{ccc|ccc} 0 & 1 & 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 2 & -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 5 & -\frac{5}{2} & 1 & 1 \end{array} \right] \begin{array}{l} R_2 - 3R_1 \\ R_3 - 5R_1 \end{array} \\ &\rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & -\frac{3}{2} & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{5} & \frac{1}{5} \end{array} \right] \begin{array}{l} R_2 \leftrightarrow R_1 \\ \frac{1}{5}R_3 \end{array} \\ &\rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & \frac{3}{5} & -\frac{2}{5} \\ 0 & 1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{5} & \frac{1}{5} \end{array} \right] R_1 - 2R_2 \end{aligned}$$

- (b) Based on your answer to part A, find  $A^{-1}$  or explain why it does not exist.

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{3}{5} & -\frac{2}{5} \\ \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

$$\text{Check: } AA^{-1} = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & \frac{3}{5} & -\frac{2}{5} \\ \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{1}{5} & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

2. Rutgers landscaping is purchasing plants for campus, and needs at least 350 plants with red flowers and at least 300 plants with white flowers. A flat of POPPIES contains 15 plants with red flowers and 20 plants with white flowers and costs \$60. A flat of GERANIUMS contains 20 plants with red flowers and 10 plants with white flowers and costs \$40. A flat of PETUNIAS contains 10 plants with red flowers and 15 plants with white flowers and costs \$30. (Note: poppies, geraniums and petunias are types of flowers.)

Set up a linear programming model <sup>for minimizing the cost</sup> of the situation described. Determine if the model is in standard form. If it is not, state what must be changed to put the model into standard form. Do **not** solve the linear programming model.

$$\text{Let } x = \# \text{ flats of POPPIES} \quad y = \# \text{ flats of GERANIUMS} \quad z = \# \text{ flats of PETUNIAS}$$

$$\text{Cost} = \text{Cost of POPPIES} + \text{cost of GERANIUMS} + \text{cost of PETUNIAS}$$

$$\text{Minimize } Z = 60x + 40y + 30z$$

$$\text{Subject to: } 15x + 20y + 10z \geq 350$$

$$20x + 10y + 15z \geq 300$$

$$x, y, z \geq 0$$

(total # of red plants)

(total # of white plants)

(can't have negative # of flats)

This is not in standard form because

① not a maximization problem (change  $z = 60x + 40y + 30z$  to  $z = -60x - 40y - 30z$ )

② Constraints aren't  $\leq$  (replace constraints with:

$$\begin{aligned} -3x - 20y - 10z &\leq -350 \quad \& \\ -20x - 10y - 15z &\leq -300 \end{aligned}$$